

# Non-Fermi-Liquid in a modified single electron transistor

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At low temperatures, a system built from a small droplet of electrons and a larger, but still finite, droplet may display non-Fermi-liquid behavior. Stabilization of a multi-channel Kondo fixed point requires fine control of the electrochemical potential in each droplet. The desired fine control can be achieved by adjusting voltages on nearby gate electrodes. We study the conditions for obtaining this type of non-Fermi-liquid behavior and discuss the experimentally-observable consequences.

Despite the presence of strong electron-electron interactions, most “metallic” conductors are well-described by Landau’s phenomenological Fermi-liquid (FL) theory [1,2]. Among the few exceptions are various types of superconductors [2], Laughlin liquids in two dimensional electron systems at strong magnetic field [3], and Luttinger liquids in one dimension [4]. In this Letter we argue that a simple configuration of two or more large electron droplets (see Fig. 1) attached to a small electron droplet can exhibit multi-channel Kondo (MCK) correlations [5], retaining non-Fermi-liquid (NFL) behavior at low temperature.

The single-channel Kondo (1CK) effect has been studied for decades in metals with magnetic impurities [6]. The same phenomenon was observed recently in semiconductor nanostructures containing no magnetic impurities: instead, in each study an electron droplet with a degenerate ground state plays the role of a magnetic impurity, and nearby electron reservoirs play the role of the surrounding normal metal [7–12].

In contrast, due to an intrinsic channel anisotropy the MCK effect may not be observable in metals with magnetic impurities [13], making it an even more intriguing phenomenon to produce in artificial nanostructures [14–17]. We will show below that in a certain configuration of electron droplets (Fig. 2), tuning the voltage on just one gate electrode can stabilize the two-channel Kondo (2CK) fixed point at low temperature. At this fixed point the conductance through the small droplet should have an anomalous power-law dependence on temperature, a manifestation of NFL behavior. The simplicity of the structure and the ability to tune system parameters hold out hope for detailed study of the NFL realm, including non-thermodynamic quantities such as transmission phase [18,19] and tunneling density of states.

To see how NFL behavior can be realized in a system of electron droplets, we first discuss a model in which a few large conducting droplets are attached to a single small droplet (see Fig. 1). A small central “pistil” droplet (denoted by  $d$ ) has a single level of energy  $\varepsilon_{ds}$ , which can be empty, or occupied by electrons of either or both spin directions  $s = \uparrow, \downarrow$ . Henceforth we refer to this droplet as *small dot*  $d$ . In the large “petal” droplets we neglect the discreteness of single-particle energy levels, while re-

taining a finite Coulomb energy. Thus, they behave as “interacting leads”; we refer to them as *large dots*.

To describe this system we use the model Hamiltonian:

$$H = \sum_{kls} \varepsilon_{lks} l_{ks}^\dagger l_{ks} + \sum_s \varepsilon_d^0 d_s^\dagger d_s + U n_{d\uparrow} n_{d\downarrow} + \sum_l u_l (n_l - \mathcal{N}_l)^2 + \sum_{ks} \left( V_{lk}^* l_{ks}^\dagger d_s + V_{lk} d_s^\dagger l_{ks} \right). \quad (1)$$

Here  $l_{ks}$  is the annihilation operator of an electron at state  $k$ , spin  $s$  and energy  $\varepsilon_{lks}$  at large dot  $l = 1, \dots, N$ ,  $d_s$  is the annihilation operator of an electron with spin  $s$  at small dot  $d$ ,  $n_l = \sum_{ks} l_{ks}^\dagger l_{ks}$  is the number operator at dot  $l$ , and  $n_{d\uparrow(\downarrow)} = d_{\uparrow(\downarrow)}^\dagger d_{\uparrow(\downarrow)}$ . The parameter  $\mathcal{N}_l$  sets the equilibrium occupancy of dot  $l$ , and  $\varepsilon_d^0$  is the bare energy of level  $d$ . Each of these can be independently tuned by adjusting potentials on nearby gates.

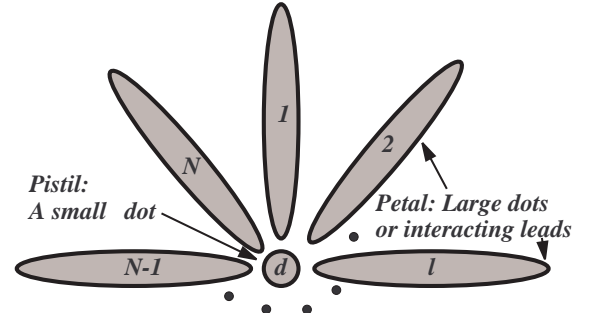


FIG. 1. A few large, but finite, conducting droplets (large dots) tunnel coupled to a small one-level droplet (dot  $d$ ).

Glazman and Raikh showed that a local site coupled to two noninteracting leads maps to a 1CK problem, in which only an even linear combination of the electron creation and annihilation operators in the two leads couples to the local spin [20]. This result generalizes straightforwardly to arbitrary number of leads in the absence of Coulomb charging ( $u_l = 0$  for all  $l$ ). The well-known 1CK system displays an interesting many-body resonance, however, at very low temperatures we can describe it simply as a FL superimposed with a resonance [21].

A nonzero  $u_l$  induces interactions among some of the transformed lead operators, substantially changing the low-temperature behavior of the system. In terms of the

original large dots the physical role of the  $u_l$  terms is clear: at low temperatures they forbid processes in which charge is ultimately transferred from one large dot to another. Spin flip events — *e.g.* when an electron hops onto the small dot and then an electron with an opposite spin hops off the small dot to the same large dot — remain possible and may lead to a MCK fixed point.

To see this, we study the case  $\varepsilon_d^0(\varepsilon_d^0 + U) < 0$ , and integrate out in the renormalization group (RG) sense all energies up to a cutoff  $\tilde{D} \sim a|\varepsilon_d| \lesssim |\varepsilon_d|, |\varepsilon_d + U|$ , with  $\varepsilon_d = \varepsilon_d^0 + \Gamma/\pi \log(aD/|\varepsilon_d|)$ . Here  $\Gamma$  is the width of level  $d$  in the absence of interactions,  $D$  is a cutoff of order the Fermi energy,  $a$  is a number of order 1, and we have assumed  $\varepsilon_{d\uparrow} = \varepsilon_{d\downarrow} \equiv \varepsilon_d$  [22]. Now we can safely perform the Schrieffer-Wolff transformation [23] and find:

$$H = \sum_{kls} \varepsilon_{lks} l_{ks}^\dagger l_{ks} + \sum_l u_l (n_l - \mathcal{N}_l)^2 + \sum_{lm,kq} J_{lm}^{kq} \left[ S^+ s_{lm}^{-kq} + S^- s_{lm}^{+kq} + 2S^z s_{lm}^{zkq} \right], \quad (2)$$

where  $S^\pm = d_{\uparrow(\downarrow)}^\dagger d_{\downarrow(\uparrow)}$ ,  $S^z = \frac{1}{2} (d_{\uparrow}^\dagger d_{\uparrow} - d_{\downarrow}^\dagger d_{\downarrow})$ ,  $s_{lm}^{\pm kq} = l_{k\uparrow(\downarrow)}^\dagger m_{q\downarrow(\uparrow)}$ ,  $s_{lm}^{zkq} = \frac{1}{2} (l_{k\uparrow}^\dagger m_{q\uparrow} - l_{k\downarrow}^\dagger m_{q\downarrow})$  and

$$J_{lm}^{kq} = V_{lk} V_{mq}^* \left[ \frac{1}{E_{\text{elec}}^{qm} - E_{\text{init}}} + \frac{1}{E_{\text{hole}}^{kl} - E_{\text{init}}} \right]. \quad (3)$$

Here  $E_{\text{init}}$  is the energy of the initial state with one electron in the small dot  $d$  and occupancy  $n_l$  ( $n_m$ ) in large dot  $l$  ( $m$ );  $E_{\text{elec}}^{qm}$  denotes the energy of an intermediate state of a process where first an electron hops from state  $q$  of dot  $m$  onto dot  $d$  and then the same electron or an electron with opposite spin hops off dot  $d$  onto state  $k$  of dot  $l$ ;  $E_{\text{hole}}^{kl}$  is the energy of the intermediate state of a process where the temporal order of the hopping events is interchanged.

Let us assume that  $\varepsilon_{l(m)qs} \approx \varepsilon_{l(m)F}$ , where  $\varepsilon_{l(m)F}$  is the energy of the last empty (occupied) state in dot  $l$  ( $m$ ), and that  $V_{mq} = V_{lk} \equiv V$ . Then using Eq. (1) to express the energies in Eq. (3) we find:

$$J_{lm}^{kq} = J_{lm} = |V|^2 \frac{U + u_m^- + u_l^+}{[U + \varepsilon_d + u_m^-] [u_l^+ - \varepsilon_d]}, \quad (4)$$

where  $u_p^\pm = u_p [1 \pm 2(n_p - \mathcal{N}_p)] \pm \varepsilon_{pF}$ , and for  $-1/2 < \mathcal{N}_p - n_p + (\mu - \varepsilon_{pF})/(2u_p) < 1/2$  there are  $n_p$  electrons in dot  $p$ , where  $p = m, l$  and  $\mu$  is the electrochemical potential of a reference reservoir. Particle hole symmetry may be absent in the large dots, so in general  $J_{lm} \neq J_{ml}$ .

The off-diagonal terms describe transfer of an electron from dot  $m$  to dot  $l$ ; at low temperature these processes are exponentially suppressed as  $J_{lm} = J_{lm}^0 e^{u_{lm}/(4T)}$  with  $u_{lm} = (u_l^+ + u_m^-)(1 - \delta_{lm})$ . Notice that  $u_{ll} = 0$ , since the charge on dot  $l$  is not changed when an electron hops from dot  $l$  onto dot  $d$  and then back to the same dot  $l$ .

At  $T < \min(u_{lm})$  only the diagonal terms of  $\tilde{J}_{lm}$  do not flow to zero. Assuming that we are not at a degeneracy point where  $u_{lm} = 0$ , an easy condition to avoid, the RG equations are identical to the MCK RG equations [24]:

$$\frac{d\tilde{J}_{ll}}{d \log(\tilde{D}/T)} = \tilde{J}_{ll}^2 - \tilde{J}_{ll} \sum_p \tilde{J}_{pp}^2. \quad (5)$$

As in the case of classic MCK, our NFL fixed point is unstable to the introduction of channel anisotropy. If one of the coupling constants is larger than the others, the corresponding channel alone screens the local spin and forms a Kondo resonance while the other channels are decoupled from the local spin. In our model we can tune  $\mathcal{N}_l$  to achieve  $J_{ll} = J$  for all  $l$ . For  $N$  large dots this requires tuning of  $N - 1$  gate potentials.

These gate potentials capacitively control the energy of the last occupied level in each large dot, so excitations in each large dot will be around a different Fermi energy. This does not modify the RG equations, but will affect certain physical properties such as the small dot density of states at finite energies. A similar situation occurs in the discussion of 2CK in a dot out of equilibrium [17,14].

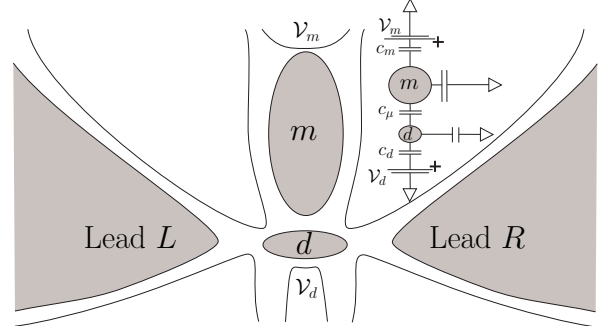


FIG. 2. A proposed realization of the two-channel Kondo (2CK) model. Two non-interacting leads ( $L$  and  $R$ ) and a large dot  $m$  are attached to a single-level small dot  $d$ . If dot  $d$  is occupied by a single electron, it can flip its spin by virtually hopping the electron onto either dot  $m$  or the leads, and then returning an electron with opposite spin to dot  $d$ . Dot  $m$  and the leads thus serve as the two distinct screening channels required to produce the 2CK effect. Crucially, when  $T$  is smaller than the charging energy of dot  $m$ , Coulomb blockade blocks transfer of electrons between the leads and dot  $m$ . Fine tuning of  $V_m$  (and/or  $V_d$ ) can equalize the coupling to the two channels, stabilizing the 2CK fixed point.

For large  $N$  it would be difficult to realize MCK experimentally, requiring tuning at least  $N - 1$  gate potentials to obtain the desired fixed point. We therefore propose a specific realization of the 2CK model (Fig. 2). In this “three leg” structure two noninteracting “free” leads  $L$  and  $R$  — effectively large dots with  $u_{L,R} = 0$  — are connected to small dot  $d$  which is in turn connected to large dot  $m$ . Dot  $m$  is not directly connected to the leads. This structure allows conductance measurements between leads  $L$  and  $R$  through dot  $d$ , and proper tuning

of parameters should cause 2CK effect to proclaim itself in this conductance.

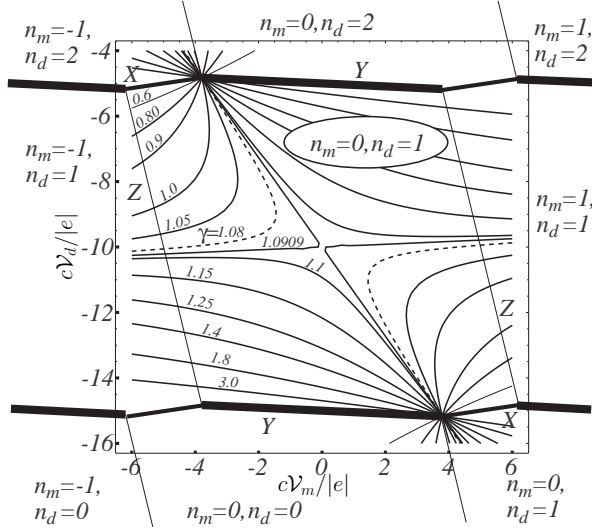


FIG. 3. The number of electrons on dots  $m$  and  $d$  as a function of the gate potentials  $\mathcal{V}_m$  and  $\mathcal{V}_d$  (see Fig. 2 and [26]). Transitions between configurations with identical  $n_m$  but different  $n_d$  (marked by the letter  $Y$ ) occur by transfer of one electron from dot  $d$  to the leads, and those in which  $n_m$  increases by one and  $n_d$  decreases by one or vice-versa ( $X$ ) occur by transfer of an electron between dots  $m$  and  $d$ . By contrast, the thin lines ( $Z$ ) across which only  $n_m$  changes are reminders that there is no direct coupling between dot  $m$  and the leads, making direct transitions across these lines difficult. The curves superimposed on the central hexagon (“2CK-lines”) map where in the  $\mathcal{V}_m, \mathcal{V}_d$  plane the two-channel Kondo (2CK) effect is realized. Each value of the coupling ratio  $\gamma \equiv \Gamma_m/\Gamma_l$  gives rise to a pair of disjoint curves, as illustrated for  $\gamma = 1.08$  (dashed) [26]. These two curves divide the hexagon into three regions with distinct low-temperature fixed points. On the curves the 2CK effect is realized and the deviation of the interlead differential conductance from its  $T \rightarrow 0, \mathcal{V}_{LR} \rightarrow 0$  limit  $G(0,0)$  is  $\propto \sqrt{\max(T, \mathcal{V}_{LR})}$ , [Eq. (9a)]. In regions adjoining the  $X$  boundaries dot  $m$  “wins”, forming a 1CK resonance with dot  $d$ , and driving  $G(T, \mathcal{V}_{LR})$  close to zero [Eq. (9b)]. In regions adjoining the  $Y$  boundaries, leads  $L$  and  $R$  “win” giving rise to Fermi-liquid behavior  $G(0,0) - G(T, \mathcal{V}_{LR}) \propto [\max(T, \mathcal{V}_{LR})]^2$ , [Eq. (9c)].

Let the potential  $\mathcal{V}_d$  on the gate near dot  $d$  be such that dot  $d$  is singly-occupied in the absence of coupling to dot  $m$ . Then if  $\Gamma_m$ , the bare coupling between dots  $d$  and  $m$ , is roughly equal to  $\Gamma_l$ , the coupling between dot  $d$  and the leads, fine-tuning the gate potential  $\mathcal{V}_m$  can stabilize the 2CK effect. To demonstrate this, we analyze the electrostatics of the circuit shown in the inset of Fig. 2.

For fixed number of electrons  $n_m$  ( $n_d$ ) on dots  $m$  ( $d$ ), the electrostatic energy of the system is:

$$\begin{aligned} E_{\text{ele}}^{n_m, n_d} &\equiv E_{\text{ele}}^{n_m, n_d}(\mathcal{V}_m, \mathcal{V}_d) \\ &= U(n_d - \mathcal{N}_d)^2 + u_m(n_d + \alpha n_m - \mathcal{N})^2, \end{aligned} \quad (6)$$

where  $U \equiv e^2/(2\tilde{C}_d) \gg u_m \equiv e^2/(2\tilde{C}_m - c_\mu^2/\tilde{C}_d)$ ,  $|e|\mathcal{N}_d \equiv$

$c_d\mathcal{V}_d$ ,  $|e|\mathcal{N} \equiv c_m\mathcal{V}_m + c_d c_\mu/\tilde{C}_m\mathcal{V}_d$  and  $\alpha \equiv c_d c_\mu/\tilde{C}_d$ . Here  $\tilde{C}_{m(d)}$  is the *total* capacitance of dot  $m(d)$ . See Fig. 2 for definitions of the other capacitances.

Since dot  $m$  is large we assume that  $\tilde{C}_m$  is much larger than all other capacitances. The number of electrons on each dot can be estimated by minimizing  $E_{\text{ele}}$  with respect to  $n_m$  and  $n_d$  [25]. In Fig. 3 we plot the number of electrons on each dot as a function of  $\mathcal{V}_m$  and  $\mathcal{V}_d$  [26].

To write down the full Hamiltonian  $\tilde{H}$  of the model system we note that there is no Coulomb blockade for transfer of electrons between  $L$  and  $R$  leads, so we can define  $l_{ks} = \cos\theta L_{ks} + \sin\theta R_{ks}$ ,  $o_{ks} = \cos\theta R_{ks} - \sin\theta L_{ks}$ ,  $\tan\theta = V_R/V_L$ ,  $V_l = \sqrt{|V_L|^2 + |V_R|^2}$ . Without loss of generality we take the coupling constants  $V_i$ ,  $i = L, R, m$ , to be real. With these definitions:

$$\begin{aligned} \tilde{H} = & \sum_{i=o,m,l;ks} \epsilon_{iks} i_{ks}^\dagger i_{ks} + \sum_s \epsilon_{ds} d_s^\dagger d_s + E_{\text{ele}}^{n_m, n_d}(\mathcal{V}_m, \mathcal{V}_d) \\ & + V_m \sum_{ks} m_{ks}^\dagger d_s + V_l \sum_{ks} l_{ks}^\dagger d_s + \text{h.c.}, \end{aligned} \quad (7)$$

which is similar to Eq. (1). To obtain a 2CK fixed point we tune  $\mathcal{V}_m$  and  $\mathcal{V}_d$  to make  $\tilde{J}_{mm}$ , the coupling of dot  $d$  to dot  $m$ , equal to  $\tilde{J}_{ll}$ , the coupling of dot  $d$  to the leads:

$$\begin{aligned} \tilde{J}_{mm}(\mathcal{V}_m, \mathcal{V}_d) &\equiv \Gamma_m \left[ \frac{1}{E_{\text{ele}}^{1,0} - E_{\text{ele}}^{0,1}} + \frac{1}{E_{\text{ele}}^{-1,2} - E_{\text{ele}}^{0,1}} \right] = \\ \tilde{J}_{ll}(\mathcal{V}_m, \mathcal{V}_d) &\equiv \Gamma_l \left[ \frac{1}{E_{\text{ele}}^{1,0} - E_{\text{ele}}^{0,2}} + \frac{1}{E_{\text{ele}}^{0,0} - E_{\text{ele}}^{0,1}} \right]. \end{aligned} \quad (8)$$

Here  $\Gamma_{m(l)} = |\mathcal{V}_{m(l)}|^2 \nu_{m(l)}$ , where  $\nu_{m(l)}$  is the density of states in dot  $m$  (the leads). Eq. (8) together with the ratio  $\gamma \equiv \Gamma_m/\Gamma_l$  defines a curve in the  $\mathcal{V}_m, \mathcal{V}_d$  plane. In Fig. 3 we show several of these “2CK-lines” for different values of  $\gamma$ . On these lines 2CK physics should be realized at low  $T$ .

The 2CK fixed point can be reached experimentally by fixing  $\mathcal{V}_d$  to give an odd number of electrons in the small dot, and fine tuning  $\mathcal{V}_m$  so that  $\tilde{J}_{mm}(\mathcal{V}_m, \mathcal{V}_d) = \tilde{J}_{ll}(\mathcal{V}_m, \mathcal{V}_d)$ . In Fig. 3 this corresponds to tuning  $\mathcal{V}_m$  until we hit the 2CK line for our given value of  $\gamma$ .

The current  $I$  between the left and right leads can be measured as a function of temperature  $T$ , and as a function of  $\mathcal{V}_{LR}$ , the bias applied between leads  $L$  and  $R$  (see Fig. 2). Drawing on the extensive literature of 2CK physics [27] we can predict the qualitative behavior of the  $I$ - $\mathcal{V}_{LR}$  curve through the dot, for different values of the gate voltages  $\mathcal{V}_m, \mathcal{V}_d$  that scan the hexagon of Fig. 3.

The 2CK-lines divide the hexagon into three different parts. On the 2CK-lines in the unitary limit —  $T, \mathcal{V}_{RL} \ll$  Kondo temperature  $T_K \cong \sqrt{U\Gamma_m} e^{-1/J_{mm}(\mathcal{V}_m, \mathcal{V}_d)}$  — the differential conductance  $G(T, \mathcal{V}_{LR}) \equiv dI/d\mathcal{V}_{LR}$  should approach its limiting value  $G(0,0)$  as

$$2CK : G(0,0) - G(T, \mathcal{V}_{LR}) \propto \sqrt{\max(\mathcal{V}_{LR}, T)}. \quad (9a)$$

In the symmetric case  $V_L = V_R$ , we get  $G(0,0) = G_K \equiv e^2/(2\pi\hbar)$ , half the maximal value of  $G(0,0)$  in the 1CK effect [10,12].

In regions of the hexagon adjoining boundaries labelled  $X$  in Fig. 3, (*i.e.* for  $\tilde{J}_{mm} > \tilde{J}_{ll}$ ), as temperature decreases the electrons in dot  $m$  screen the spin of dot  $d$ , while the leads are decoupled. In the RG sense  $\tilde{J}_{ll}$  flows to zero, so that dot  $m$  “wins” over the leads and forms a 1CK state with dot  $d$ . In that case  $dI/dV_{LR}$  is small and given by

$$\text{Large dot wins: } G(T, V_{LR}) \propto [\max(V_{LR}, T)]^2. \quad (9b)$$

In contrast, in regions of the hexagon adjoining boundaries labelled  $Y$  in Fig. 3 (*i.e.* for  $\tilde{J}_{mm} < \tilde{J}_{ll}$ ), dot  $m$  decouples from dot  $d$  at low temperature, leaving the leads to form a 1CK resonance with dot  $d$  and

$$\text{Leads win: } G(0,0) - G(T, V_{LR}) \propto [\max(V_{LR}, T)]^2, \quad (9c)$$

where  $G(0,0) = 2G_K$  for  $V_L = V_R$ .

At sufficiently low temperature the finite level spacing  $\Delta_m$  in dot  $m$  will cut off the RG flow of the coupling constants [28]. We cannot make  $\Delta_m$  infinitesimal as we must retain a finite Coulomb blockade energy  $u_m > kT$ . However, the ratio between charging energy and level spacing can be made large, allowing 2CK behavior to be observed over a significant temperature range before the system finally flows to the 1CK fixed point.

In conclusion, if a small dot is coupled to multiple electron reservoirs (large dots) Coulomb blockade suppresses inter-reservoir charge transfer at low temperatures. In the simplest version of this system, electrostatic gates provide the tunability needed to stabilize a 2CK fixed point, resulting in observable NFL behavior. Softer versions of suppressing inter-reservoir tunneling could also work in place of Coulomb blockade. For example, the reservoirs could be conductors with large impedance [29], one-dimensional Luttinger liquids [30] or conductors with strongly-interacting charge carriers. Finally, while the channel asymmetry parameter is relevant in the RG sense, for realistically well-matched channel couplings we expect that the system will remain near the 2CK fixed point, and will show NFL behavior, over a wide range of temperatures.

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